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ABSTRACT

This paper describes three methods that can be utilized in teaching for meaning in mathematics. These three methods are based on learning theories. Although there may not be consensus of what constitutes meaningful learning in the study of mathematics, there are problems with meaningless mathematics that are widely agreed upon by educators. Teaching methods that contribute to students being able to ground mathematical concepts meaningfully are widely needed. The expectation is that students will be able to apply learned concepts, skills and generalizations when dealing with everyday situations and that they will be able to solve real world problems. (Author)

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Teaching for Meaning in Mathematics

by
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TEACHING FOR MEANING IN MATHEMATICS

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This paper describes three methods that can be utilized in teaching for meaning in mathematics. These three methods are based on learning theories. Although there may not be a consensus of what constitutes meaningful learning in the study of mathematics, there are problems with meaningless mathematics that are widely agreed upon by educators. Teaching methods that contribute to students being able to ground mathematical concepts meaningfully are widely needed. The expectation is that students will be able to apply learned concepts, skills and generalizations when dealing with everyday situations and that they will be able to solve real world problems.

The literature reveals that students' mathematical performance is deficient and that they have difficulty in mathematics. We often hear statements such as, algebra is a "gatekeeper" course (Kamii, 1990). That is, those students who have difficulty with algebra tend not to elect higher level mathematics courses. Another such statement is, mathematics is a "critical filter" barring students, particularly women and Blacks, from entry into universities, from selecting scientific or technical majors, or from selecting other majors that require more than the minimum mathematics (Sells, 1973; 1976).

Why is algebra a gatekeeper course? Why is mathematics a critical filter? Why do students have difficulty in mathematics and make common errors? These are questions that educators, particularly mathematics educators, would like to answer. The answers to these questions may lie with learning theories as they relate to meaning. In many instances learning and meaning are linked. Some results are as follows.

- Meanings facilitate learning
- Meanings increase the chances of transfer
- Meaningful mathematics is better retained
- Mathematics can function in intelligent living only when it is understood
- Meaningful learning safeguards students from answers that are mathematically absurd

In addition to these statements, Weaver and Suydam (1972) reviewed the literature and reported the results of meaningfully taught content versus non-meaningfully taught content. Much of the research supported the "meaning" as opposed to the "non-meaning." Although there may not be a consensus of what constitutes meaningful learning in the study of mathematics, there are problems with meaningless mathematics that are widely agreed upon by educators. So "teaching for meaning in mathematics" has much support.

Presented here are three methods that can be used as one teaches for meaning in mathematics. Each method is based on a learning theory and is utilized to remediate common errors of the type

$$(a+b)^c = a^c + b^c \quad \text{and} \quad \sqrt[c]{a+b} = \sqrt[c]{a} + \sqrt[c]{b}\ .$$

The theories describe the behavior of the expert. So the object of each teaching method is to provide alternative experiences for novices intending to lead to more expert-like performance,

based on the three differing theories of expertise described.
The methods are

- The ML Method, based on the Meaning Theory of Learning
- The PL Method, based on the Procedural Theory of Learning
- The ISL Method, based on the Implicit Structure Theory of Learning

(Bates, 1994/1995).

The meaning theory of learning (ML) hypothesizes that experts have lots of rich meanings connected with procedures and symbols of mathematics but novices lack these connections. Thus, according to this theory, the errors

$$(a + b)^c = a^c + b^c \quad \text{and} \quad \sqrt[c]{a + b} = \sqrt[c]{a} + \sqrt[c]{b}$$

are the result of a lack of connections and references for expressions and symbols involved. Such connections serve to help students remember rules and to constrain students' arbitrary mathematics inventions, according to the meaning theory of learning (Brownell, 1947).

The ML method is a teaching technique whereby the teacher provides a variety of rich semantic experiences. In the case of remediating errors of the type,

$$(a + b)^c = a^c + b^c \quad \text{and} \quad \sqrt[c]{a + b} = \sqrt[c]{a} + \sqrt[c]{b} ,$$

some of the experiences designed for the students can include the following.

- Using numerical instances to evaluate whether proposed rules are correct or incorrect:
- Using axiomatic methods to establish equivalent expressions; and
- Constructing geometrical models of expressions to verify equivalences.

These type experiences provide connections for the students and thus prevent the errors.

The procedural theory of learning (PL) hypothesizes that adept problem solvers have technical proficiency in memorizing and applying mechanical rules. They are able to match the structure of the problem with the structure of the rule that is to be applied. According to Matz (1980), students' errors are the result of unsuccessful attempts to employ extrapolation techniques to adapt previously acquired rules to new situations. The adept problem solver has learned to constrain extrapolations more successfully. Thus, the errors result from the students employing inappropriate extrapolations.

The PL method is a teaching technique whereby the teacher involves the students in the study of the structure of rules. In the case of remediating errors of the type,

$$(a + b)^c = a^c + b^c \quad \text{and} \quad \sqrt[c]{a + b} = \sqrt[c]{a} + \sqrt[c]{b} ,$$

some of the activities designed for the students can include the following.

- Reviewing the textbook rules and an illustration these rules with numerical instances
- Identifying expressions where these rules can or can not be applied.
- Generating possible new rules for situations in which the given rules are inapplicable, and assessing the validity of these new rules.
- Warning about overgeneralizing given rules without verifications can be stressed by the teacher.
- A contrived rule system can be used to further the students' procedural competence.

These experiences help the students recognize and understand the structure of expressions and thus they are able to apply the rule when applicable.

The implicit structure theory of learning (ISL) hypothesizes that people skillful at elementary algebra have developed an unconscious abstract rule system that underlies their successful performance. According to this theory, students enter the classroom with nascent abstract algebraic rule structures on which to build; and then they begin to sort out the conditions under which the particular structures apply. That is, the students are inductively and unconsciously experimenting to fashion an abstract "grammar" of algebra. According to this theory, errors occur as the students experiment with an abstract rule, such as the abstract distributive rule,

(a * b) @ c = (a @ c) * (b @ c),

in search of the maximally permissible context in which it applies in algebra (Kirshner, 1987).

The ISL method is a teaching technique whereby the teacher provides experiences that enables students to determine the constraints of distributivity as in the case of remediating the errors of the type,

$$(a+b)^c = a^c + b^c \quad \text{and} \quad \sqrt[c]{a+b} = \sqrt[c]{a} + \sqrt[c]{b} \; .$$

The experiences for the students can include the following.

- Identifying examples of the distributive structure in natural language
- Generating correct and incorrect rules of distributivity in algebra to compare and contrast; and
- Identifying the operation levels in order to provide an additional catalyst that would assist students in determining the constraints of distributivity.

These experiences help the students determine the constraints of distributivity.

Each of the methods, the ML, the PL and the ISL, based on a learning theory, illustrates what is involved in teaching for meaning in mathematics. Each theory defines meaningful learning in a different manner but the payoff of teaching for meaning is the same. Teaching for meaning in mathematics is a goal to enable students to understand what they are learning so that they can apply learned concepts in dealing with everyday situations and real world problems.

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